

RobustOptThirdTrial

June 23, 2024

0.0.1 Robust Pricing Optimization - Single Product - Price Chosen within a Continuous Interval

Packages import

```
[1]: import numpy as np
import pandas as pd
import gurobipy as grb
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.metrics import mean_absolute_error
import statsmodels.api as sm
from scipy import stats as st
```

Firstly we generate a sample from a selected linear demand function (alpha and beta chosen arbitrarily), moreover we specify the production cost per unit that represents a lower bound for our decision variable, the price.

```
[2]: np.random.seed(42)
# number of samples
n = 20
var = 1
# production cost
prod_cost = 10
# demand function's real slope and intercept
alpha_real = 100
beta_real = -4
# lower and upper bounds
lower_p = prod_cost
upper_p = -alpha_real / beta_real # upper bound to maintain the demand greater or
→ equal to zero, below this value it will be negative

eps = np.random.normal(0, var, n) # add some noise
price = np.random.uniform(lower_p, upper_p, n)
demand = alpha_real + beta_real * price + eps
```

Robust solution nominal problem The max-min optimization is actually the simple max because there is no uncertainty in the nominal case, all the coefficients are known. The optimal

objective value obtained will be the benchmark to which compare the other results since it would be the best revenue possible.

```
[3]: try:
      # create the model
      model = grb.Model()
      # create decision variables = price
      p = model.addVar(ub=upper_p, lb=lower_p, vtype='C', name='p')
      # set objective function
      expr = (alpha_real + beta_real*p)*p
      model.setObjective(expr, sense=grb.GRB.MAXIMIZE)
      # solve the problem
      model.optimize()
      for v in model.getVars():
          print('%s %g' % (v.varName, v.x))
          print('Obj: %g' % model.objVal)
      except grb.GurobiError as e:
          print('Error code' + str(e))
      except AttributeError:
          print('Encountered an attribute error')
      obj_nominal = model.objVal
```

Restricted license - for non-production use only - expires 2025-11-24
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x0a88155f

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [8e+00, 8e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)

Optimal objective 6.25000000e+02

p 12.5

Obj: 625

```
[4]: # DATAFRAME FOR LINEAR REGRESSION
data = pd.DataFrame({'price': price, 'demand': demand})
print(data)
```

	price	demand
0	16.841050	32.884158
1	21.777639	12.820310
2	12.995107	48.343417
3	17.713517	29.907449
4	18.886219	24.338049
5	10.696756	57.095907
6	19.113173	24.336915
7	12.557862	50.152270
8	10.975774	55.862167
9	24.233283	3.338148
10	24.484480	1.830369
11	22.125960	11.263294
12	14.569207	41.844155
13	11.465082	53.183033
14	20.263495	18.083559
15	16.602287	33.309707
16	11.830574	52.171290
17	17.427654	30.446509
18	10.515828	57.482677
19	23.639806	4.734624

Train - Test split to partition the data records and evaluate linear regression model on the test set.
Plot the points and the regression line to see how good the linear function fits the data.

```
[5]: #X = pd.DataFrame({'price': data['price'].copy()})
#y = pd.DataFrame({'demand': data['demand'].copy()})
#np.random.seed(42)
# TRAIN TEST SPLIT
#X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2)
#returns = np.linalg.lstsq(X_train, y_train, rcond=None)
#params = returns[0]
# REGRESSION COEFFICIENTS ESTIMATIONS
#alpha = params[1]
#beta = params[0]
#y_pred = alpha + beta * X_test.loc[:, 'price']
#mae = mean_absolute_error(y_pred, y_test)
#r2 = 1 - returns[1] / (y_train.size * y_train.var())
#print(r2) # r2 = 0.997208
# PLOT
#fig, ax = plt.subplots(figsize=(8,10))
#ax.scatter(X_test.loc[:, 'price'], y_test, c='red', label='Data Points')
#ax.plot(X_test.loc[:, 'price'], y_pred, c='blue', label='Regression Line')
#ax.set_xlabel('Price')
```

```
#ax.set_ylabel('Demand')
#ax.set_title('Linear relationship between price and demand')
#plt.grid()
#plt.legend()
#plt.show()
```

We now train the model on the entire dataset and compute regression parameters (alpha, beta) and their respective confidence intervals (level 5%). Those ranges will be used to apply uncertainty set sampling technique.

```
[6]: X = pd.DataFrame({'price': data['price'].copy()})
y = pd.DataFrame({'demand': data['demand'].copy()})
X['constant'] = 1
# LINEAR REGRESSION
np.random.seed(42)
returns = np.linalg.lstsq(X, y, rcond=None)
params = returns[0]
alpha = params[1]
beta = params[0]
sum_sq_residuals = returns[1] # sum of squared residuals
dof = X.shape[0] - len(params) # degrees of freedom
mse = sum_sq_residuals / dof # mean squared error
cov = mse * np.diagonal(np.linalg.inv(X.T @ X)) # covariance matrix
se = np.sqrt(cov) # standard errors
conf_level = 0.05 # significance level
t = st.t.ppf(1 - (conf_level / 2), dof) # t-distribution quantile function
delta = t * se # margin of error
# confidence interval 95%
conf_int_alpha = np.array([[alpha - delta[0]], [alpha + delta[0]]])
conf_int_beta = np.array([[beta - delta[1]], [beta + delta[1]]])
```

Uncertainty set construction through uniform sampling, we extract n realizations of uncertain parameters alpha and beta from their interval estimates.

```
[7]: np.random.seed(42)
n_samples = 50
alpha_sim = np.random.uniform(conf_int_alpha[0, 0], conf_int_alpha[1, 0],
    ↪size=n_samples)
beta_sim = np.random.uniform(conf_int_beta[0, 0], conf_int_beta[1, 0],
    ↪size=n_samples)
```

Now we define some attributes for the uncertainty set: mean, standard deviation, maximum and minimum of scenarios

```
[8]: alpha_min = min(alpha_sim)
alpha_max = max(alpha_sim)
beta_min = min(beta_sim)
beta_max = max(beta_sim)
```

```
alpha_mean = sum(alpha_sim)/n_samples
beta_mean = sum(beta_sim)/n_samples
alpha_std = np.sqrt(sum((alpha_sim-alpha_mean)**2)/(n_samples-1))
beta_std = np.sqrt(sum((beta_sim-beta_mean)**2)/(n_samples-1))
```

Budget of uncertainty indicates how great will be the conservatism of the optimization problem's solution. We create an array of 11 values of delta to be tested

```
[9]: delta_min = 0
delta_max_alpha = max(np.abs(alpha_min-alpha_mean)/alpha_std, np.
    ↳abs(alpha_max-alpha_mean)/alpha_std)
delta_max_beta = max(np.abs(beta_min-beta_mean)/beta_std, np.
    ↳abs(beta_max-beta_mean)/beta_std)
delta_max = max(delta_max_alpha, delta_max_beta)
delta_test = np.linspace(delta_min, delta_max, num=10)
```

```
[10]: print(delta_test)
```

```
[0.          0.20153702 0.40307405 0.60461107 0.8061481  1.00768512
 1.20922215 1.41075917 1.6122962  1.81383322]
```

To understand how uncertainty affects the objective function value, we solve the optimization problem many times extracting each iteration with uniform distribution the coefficients estimations.

```
[11]: # Monte Carlo simulations
np.random.seed(42)
n_scenarios = 50
alpha_scen = np.random.uniform(alpha_min, alpha_max, size=n_scenarios)
beta_scen = np.random.uniform(beta_min, beta_max, size=n_scenarios)
obj_val = []
for i in range(n_scenarios):
    model = grb.Model()
    p = model.addVar(ub=upper_p, lb=lower_p, vtype='C', name='p')
    expr = (alpha_scen[i] + beta_scen[i]*p)*p
    model.setObjective(expr, sense=grb.GRB.MAXIMIZE)
    model.optimize()
    obj_val.append(model.objVal)
```

```
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))
```

```
CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
```

```
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads
```

```
Optimize a model with 0 rows, 1 columns and 0 nonzeros
```

```
Model fingerprint: 0x1c543338
```

```
Model has 1 quadratic objective term
```

Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [6e+00, 6e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)

Optimal objective 7.77938790e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xeaac3c91

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)

Optimal objective 7.05737305e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xc4d403fb

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]

Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.01s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 7.66191987e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x71cbdfdb
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 7.48483667e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x30befcf5
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [8e+00, 8e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 6.49186884e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x8ca703c4

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [7e+00, 7e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 7.58360925e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x73ed3de6

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [9e+00, 9e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.01s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.29263621e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x526ee568

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [9e+00, 9e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)

Optimal objective 5.51549747e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xb6879f96

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.01s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.21626716e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x168b4956

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [9e+00, 9e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.79653946e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x707446d3

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [8e+00, 8e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.93814780e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xb6ac54e4
Model has 1 quadratic objective term
Coefficient statistics:
Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [9e+00, 9e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 5.67790910e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0xf4ae75c6
Model has 1 quadratic objective term
Coefficient statistics:
Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)
Optimal objective 7.24308202e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x8fec6dac
Model has 1 quadratic objective term
Coefficient statistics:
Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]
QObjective range [8e+00, 8e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 5.86417755e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x83dd8866
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [9e+00, 9e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 5.69067418e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x5611583f
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [8e+00, 8e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 6.33676044e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0xd1b2b9a7
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[1e+02, 1e+02]
QObjective range	[9e+00, 9e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 5.39763040e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x8b447c79
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[1e+02, 1e+02]
QObjective range	[7e+00, 7e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 7.14499998e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xe765e479

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[1e+02, 1e+02]
QObjective range	[9e+00, 9e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.01s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 5.26973587e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xd167ff4d

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[1e+02, 1e+02]
QObjective range	[6e+00, 6e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)
Optimal objective 7.85046847e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xa365c85a

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [7e+00, 7e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)

Optimal objective 7.04307022e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x5d2c6465

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [9e+00, 9e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.51365150e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x7c70abcc

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [1e+01, 1e+01]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)

Optimal objective 5.14042663e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xa41ff808

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [7e+00, 7e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 7.18948040e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x7bf35310

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 6.82602027e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x697bf6d3

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 6.90162794e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x0744fb99

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]

Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.04s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.05 seconds (0.00 work units)
Optimal objective 7.03431480e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0xe47e9afe
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [9e+00, 9e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 5.26958961e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x570359a3
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [8e+00, 8e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.87240391e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x1fb384d8

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [9e+00, 9e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.34554614e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x145f3777

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [7e+00, 7e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 7.36535240e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x098a11d3

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [8e+00, 8e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 6.56615059e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xbbb13dc8

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [9e+00, 9e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.80234286e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xadac6fdd

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [9e+00, 9e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)

Optimal objective 5.25403546e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xe95cab4b

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [9e+00, 9e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.76655127e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x24f24bf4
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [9e+00, 9e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 5.79729436e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xcb4d79ba
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 6.89734766e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x033b7e08
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]
QObjective range [8e+00, 8e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 6.60759671e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0xdf2815c7
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [7e+00, 7e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]
Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)
Optimal objective 7.45697367e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x4e31737d
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [8e+00, 8e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 6.15176582e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0x85ff329a
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[1e+02, 1e+02]
QObjective range	[9e+00, 9e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 5.35360441e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros
Model fingerprint: 0xc5e71790
Model has 1 quadratic objective term
Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[1e+02, 1e+02]
QObjective range	[7e+00, 7e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Presolve removed 0 rows and 1 columns
Presolve time: 0.02s
Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)
Optimal objective 6.84694450e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x38786aae

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[1e+02, 1e+02]
QObjective range	[7e+00, 7e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 6.99680863e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xfe327d79

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range	[0e+00, 0e+00]
Objective range	[1e+02, 1e+02]
QObjective range	[8e+00, 8e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)
Optimal objective 6.39696342e+02
Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]
Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xa33297db

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [7e+00, 7e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 7.03406969e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xd6806cb4

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [8e+00, 8e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 6.21074487e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0x84fa9712

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [8e+00, 8e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.03 seconds (0.00 work units)

Optimal objective 6.28388032e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xa3b2f391

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]

Objective range [1e+02, 1e+02]

QObjective range [8e+00, 8e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 6.03914670e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0
(22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set
[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xc6ac3792

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [1e+01, 1e+01]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.17913558e+02

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 0 rows, 1 columns and 0 nonzeros

Model fingerprint: 0xc080f0d4

Model has 1 quadratic objective term

Coefficient statistics:

Matrix range [0e+00, 0e+00]
Objective range [1e+02, 1e+02]
QObjective range [9e+00, 9e+00]
Bounds range [1e+01, 3e+01]
RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Presolve time: 0.02s

Presolve: All rows and columns removed

Barrier solved model in 0 iterations and 0.02 seconds (0.00 work units)

Optimal objective 5.33137186e+02

The value of simulation is then computed as the average of the optimal objective values for each scenario under the deterministic threshold

```
[12]: obj_val = np.array(obj_val)
      sum(obj_val[obj_val < obj_nominal])/sum(obj_val < obj_nominal)
```

```
[12]: 558.7473046466484
```

```
[13]: fig, ax = plt.subplots(figsize=(8,5))
      ax.scatter(np.arange(0, sum(obj_val < obj_nominal)), obj_val[obj_val <
      ↳obj_nominal], color='red', label='MonteCarloObjectives')
      ax.hlines(xmin=0, xmax=sum(obj_val < obj_nominal), y=obj_nominal, color='black',
      ↳linestyle='--', label='DeterministicObjective')
```

```
plt.grid()
plt.legend()
plt.savefig('MonteCarlosimulations.png')
plt.show()
```

Now we see the difference with the robust approach, assuming a budget of uncertainty delta for the intercept and the slope. Modifying those quantities we will change the solution's level of conservatism.

```
[14]: # ROBUST PROBLEM FORMULATION THROUGH STRONG DUALITY
rob_obj_val = []
for i in range(len(delta_test)):
    robust_model = grb.Model()
    z_r = robust_model.addVar(vtype='C', name='z_r')
    price_r = robust_model.addVar(lb=lower_p, ub=upper_p, vtype='C',
    ↪name='price_r')
    mu_r = robust_model.addVars(2, lb=0, vtype='C', name='mu_r')
    lambda_r = robust_model.addVars(2, lb=0, vtype='C', name='lambda_r')
    gamma_r = robust_model.addVars(2, lb=0, vtype='C', name='gamma_r')
    teta_r = robust_model.addVars(2, lb=0, vtype='C', name='teta_r')
    robust_model.setObjective(z_r, sense=grb.GRB.MAXIMIZE)
    robust_model.addConstr(mu_r[0]*alpha_min - mu_r[1]*alpha_max +
    ↪gamma_r[0]*beta_min - gamma_r[1]*beta_max -
    ↪lambda_r[0]*(beta_mean + delta_test[i]*beta_std) +
    ↪lambda_r[1]*(beta_mean - delta_test[i]*beta_std) -
    ↪teta_r[0]*(alpha_mean + delta_test[i]*alpha_std) +
    ↪teta_r[1]*(alpha_mean - delta_test[i]*alpha_std) >= z_r, name='c1')
    robust_model.addConstr(price_r - mu_r[0] + mu_r[1] + teta_r[0] - teta_r[1]
    ↪== 0, name='c2')
    robust_model.addConstr(price_r**2 - gamma_r[0] + gamma_r[1] + lambda_r[0] -
    ↪lambda_r[1] == 0, name='c3')
    # set non convexity parameter to 2 in such a way that gurobi can accept our
    ↪constraints formulation
    robust_model.params.NonConvex = 2
    robust_model.optimize()
    rob_obj_val.append(robust_model.objVal)
```

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0x65ca6806

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range [1e+00, 1e+02]

QMatrix range [1e+00, 1e+00]

QLMatrix range [1e+00, 1e+00]

Objective range [1e+00, 1e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Continuous model is non-convex -- solving as a MIP

Presolve removed 0 rows and 1 columns

Presolve time: 0.00s

Presolved: 5 rows, 9 columns, 27 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 9 continuous, 0 integer (0 binary)

Found heuristic solution: objective 621.9875788

Root relaxation: objective 7.456835e+02, 6 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	745.68346	0	1	621.98758	745.68346	19.9%	-	0s
0	0	671.67178	0	1	621.98758	671.67178	7.99%	-	0s
0	0	634.66594	0	1	621.98758	634.66594	2.04%	-	0s
0	0	625.02307	0	1	621.98758	625.02307	0.49%	-	0s
0	0	622.80806	0	1	621.98758	622.80806	0.13%	-	0s
0	0	622.15645	0	1	621.98758	622.15645	0.03%	-	0s
0	2	622.15645	0	1	621.98758	622.15645	0.03%	-	0s

Explored 5 nodes (17 simplex iterations) in 0.08 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 621.988

Optimal solution found (tolerance 1.00e-04)

Best objective 6.219875788034e+02, best bound 6.220360061589e+02, gap 0.0078%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0xf1fdbf54

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range	[1e+00, 1e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Continuous model is non-convex -- solving as a MIP

Presolve time: 0.00s

Presolved: 5 rows, 10 columns, 29 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 10 continuous, 0 integer (0 binary)

Found heuristic solution: objective 605.6759149

Root relaxation: objective 7.187500e+02, 6 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	718.75001	0	1	605.67591	718.75001	18.7%	-	0s
0	0	652.80974	0	1	605.67591	652.80974	7.78%	-	0s
0	0	619.83960	0	1	605.67591	619.83960	2.34%	-	0s
0	0	607.42421	0	1	605.67591	607.42421	0.29%	-	0s
0	0	606.40679	0	1	605.67591	606.40679	0.12%	-	0s
0	0	605.89808	0	1	605.67591	605.89808	0.04%	-	0s
0	2	605.89808	0	1	605.67591	605.89808	0.04%	-	0s

Explored 5 nodes (17 simplex iterations) in 0.07 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 605.676

Optimal solution found (tolerance 1.00e-04)

Best objective 6.056759149016e+02, best bound 6.056906325026e+02, gap 0.0024%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0x65ee5f00

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range	[1e+00, 1e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Continuous model is non-convex -- solving as a MIP

Presolve time: 0.00s

Presolved: 5 rows, 10 columns, 29 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 10 continuous, 0 integer (0 binary)

Found heuristic solution: objective 590.1944629

Root relaxation: objective 6.918166e+02, 6 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	691.81657	0	1	590.19446	691.81657	17.2%	-	0s
0	0	633.94770	0	1	590.19446	633.94770	7.41%	-	0s
0	0	605.01327	0	1	590.19446	605.01327	2.51%	-	0s
0	0	590.54605	0	1	590.19446	590.54605	0.06%	-	0s
0	0	590.36588	0	1	590.19446	590.36588	0.03%	-	0s
0	0	590.27579	0	1	590.19446	590.27579	0.01%	-	0s
0	2	590.27579	0	1	590.19446	590.27579	0.01%	-	0s

Explored 5 nodes (22 simplex iterations) in 0.10 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 590.194

Optimal solution found (tolerance 1.00e-04)

Best objective 5.901944629256e+02, best bound 5.902236239921e+02, gap 0.0049%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0xf0958c49

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range	[1e+00, 1e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Continuous model is non-convex -- solving as a MIP

Presolve time: 0.00s

Presolved: 5 rows, 10 columns, 29 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 10 continuous, 0 integer (0 binary)

Found heuristic solution: objective 575.4814134

Root relaxation: objective 6.648831e+02, 6 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	664.88313	0	1	575.48141	664.88313	15.5%	-	0s
0	0	615.08567	0	1	575.48141	615.08567	6.88%	-	0s
0	0	590.18694	0	1	575.48141	590.18694	2.56%	-	0s
0	0	577.73757	0	1	575.48141	577.73757	0.39%	-	0s
0	0	576.35980	0	1	575.48141	576.35980	0.15%	-	0s
0	0	575.67092	0	1	575.48141	575.67092	0.03%	-	0s
0	2	575.67092	0	1	575.48141	575.67092	0.03%	-	0s

Explored 7 nodes (20 simplex iterations) in 0.07 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 575.481

Optimal solution found (tolerance 1.00e-04)

Best objective 5.754814133892e+02, best bound 5.755206496854e+02, gap 0.0068%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0x82422c5c

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range	[1e+00, 1e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Continuous model is non-convex -- solving as a MIP

Presolve time: 0.00s

Presolved: 5 rows, 10 columns, 29 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 10 continuous, 0 integer (0 binary)

Found heuristic solution: objective 561.4809439

Root relaxation: objective 6.379497e+02, 6 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	637.94969	0	1	561.48094	637.94969	13.6%	-	0s
0	0	596.22363	0	1	561.48094	596.22363	6.19%	-	0s
0	0	575.36060	0	1	561.48094	575.36060	2.47%	-	0s
0	0	564.92909	0	1	561.48094	564.92909	0.61%	-	0s
0	0	562.35373	0	1	561.48094	562.35373	0.16%	-	0s
0	0	561.69363	0	1	561.48094	561.69363	0.04%	-	0s
0	2	561.69363	0	1	561.48094	561.69363	0.04%	-	0s

Explored 5 nodes (22 simplex iterations) in 0.06 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 561.481

Optimal solution found (tolerance 1.00e-04)

Best objective 5.614809438720e+02, best bound 5.614967694300e+02, gap 0.0028%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0xb458b54e

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range	[1e+00, 1e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Continuous model is non-convex -- solving as a MIP

Presolve time: 0.00s

Presolved: 5 rows, 10 columns, 29 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 10 continuous, 0 integer (0 binary)

Found heuristic solution: objective 548.1425112

Root relaxation: objective 6.110162e+02, 6 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node			Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time
0	0	611.01625	0	1	548.14251	611.01625	11.5%	-	0s
0	0	577.36160	0	1	548.14251	577.36160	5.33%	-	0s
0	0	560.53427	0	1	548.14251	560.53427	2.26%	-	0s
0	0	552.12061	0	1	548.14251	552.12061	0.73%	-	0s
0	0	548.34765	0	1	548.14251	548.34765	0.04%	-	0s
0	0	548.23918	0	1	548.14251	548.23918	0.02%	-	0s
0	2	548.23918	0	1	548.14251	548.23918	0.02%	-	0s

Explored 7 nodes (19 simplex iterations) in 0.08 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 548.143

Optimal solution found (tolerance 1.00e-04)

Best objective 5.481425112479e+02, best bound 5.481535260127e+02, gap 0.0020%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0x7f950e10

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range	[1e+00, 1e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Continuous model is non-convex -- solving as a MIP

Presolve time: 0.00s

Presolved: 5 rows, 10 columns, 29 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 10 continuous, 0 integer (0 binary)

Found heuristic solution: objective 535.4202420

Root relaxation: objective 5.840828e+02, 6 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node				Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time	
0	0	584.08281	0	1	535.42024	584.08281	9.09%	-	0s	
0	0	558.49956	0	1	535.42024	558.49956	4.31%	-	0s	
0	0	545.70794	0	1	535.42024	545.70794	1.92%	-	0s	
0	0	539.31213	0	1	535.42024	539.31213	0.73%	-	0s	
0	0	536.11422	0	1	535.42024	536.11422	0.13%	-	0s	
0	0	535.67106	0	1	535.42024	535.67106	0.05%	-	0s	
0	0	535.67106	0	1	535.42024	535.67106	0.05%	-	0s	
0	2	535.67106	0	1	535.42024	535.67106	0.05%	-	0s	

Explored 7 nodes (20 simplex iterations) in 0.08 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 535.42

Optimal solution found (tolerance 1.00e-04)

Best objective 5.354202420009e+02, best bound 5.354515919629e+02, gap 0.0059%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set

[SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0x6976bb8f

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range	[1e+00, 1e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Continuous model is non-convex -- solving as a MIP

Presolve time: 0.00s

Presolved: 5 rows, 10 columns, 29 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 10 continuous, 0 integer (0 binary)

Found heuristic solution: objective 523.2724051

Root relaxation: objective 5.571494e+02, 6 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node				Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap	It/Node	Time	
0	0	557.14936	0	1	523.27241	557.14936	6.47%	-	0s	
0	0	539.63753	0	1	523.27241	539.63753	3.13%	-	0s	
0	0	530.88161	0	1	523.27241	530.88161	1.45%	-	0s	
0	0	526.50365	0	1	523.27241	526.50365	0.62%	-	0s	
0	0	524.31467	0	1	523.27241	524.31467	0.20%	-	0s	

Explored 1 nodes (10 simplex iterations) in 0.06 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 523.272

Optimal solution found (tolerance 1.00e-04)

Best objective 5.232724051268e+02, best bound 5.233198758379e+02, gap 0.0091%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0x6b666e0c

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range	[1e+00, 1e+02]
QMatrix range	[1e+00, 1e+00]
QLMatrix range	[1e+00, 1e+00]
Objective range	[1e+00, 1e+00]
Bounds range	[1e+01, 3e+01]
RHS range	[0e+00, 0e+00]

Continuous model is non-convex -- solving as a MIP

Presolve time: 0.00s

Presolved: 5 rows, 10 columns, 29 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 10 continuous, 0 integer (0 binary)

Found heuristic solution: objective 512.7643950

Root relaxation: objective 5.328042e+02, 5 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node				Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	IntInf		Incumbent	BestBd	Gap	It/Node	Time
0	0	532.80417	0	1		512.76440	532.80417	3.91%	-	0s
0	0	522.59326	0	1		512.76440	522.59326	1.92%	-	0s
0	0	517.48780	0	1		512.76440	517.48780	0.92%	-	0s
0	0	514.93507	0	1		512.76440	514.93507	0.42%	-	0s
0	0	513.65870	0	1		512.76440	513.65870	0.17%	-	0s
0	0	513.02052	0	1		512.76440	513.02052	0.05%	-	0s
0	2	513.02052	0	1		512.76440	513.02052	0.05%	-	0s

Explored 7 nodes (19 simplex iterations) in 0.09 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 512.764

Optimal solution found (tolerance 1.00e-04)

Best objective 5.127643950037e+02, best bound 5.127649536111e+02, gap 0.0001%

Set parameter NonConvex to value 2

Gurobi Optimizer version 11.0.1 build v11.0.1rc0 (win64 - Windows 11.0 (22631.2))

CPU model: 11th Gen Intel(R) Core(TM) i5-11300H @ 3.10GHz, instruction set [SSE2|AVX|AVX2|AVX512]

Thread count: 4 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 2 rows, 10 columns and 14 nonzeros

Model fingerprint: 0x561ad007

Model has 1 quadratic constraint

Coefficient statistics:

Matrix range [1e+00, 1e+02]

QMatrix range [1e+00, 1e+00]

QLMatrix range [1e+00, 1e+00]

Objective range [1e+00, 1e+00]

Bounds range [1e+01, 3e+01]

RHS range [0e+00, 0e+00]

Presolve removed 0 rows and 1 columns

Continuous model is non-convex -- solving as a MIP

Presolve removed 0 rows and 1 columns

Presolve time: 0.00s

Presolved: 5 rows, 9 columns, 27 nonzeros

Presolved model has 1 bilinear constraint(s)

Variable types: 9 continuous, 0 integer (0 binary)

Found heuristic solution: objective 512.7643950

Root relaxation: objective 5.328042e+02, 5 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node				Objective Bounds			Work	
Expl	Unexpl	Obj	Depth	Int	Inf	Incumbent	BestBd	Gap	It/Node	Time
0	0	532.80417	0	1	512.76440	532.80417	532.80417	3.91%	-	0s
0	0	522.59326	0	1	512.76440	522.59326	522.59326	1.92%	-	0s
0	0	517.48780	0	1	512.76440	517.48780	517.48780	0.92%	-	0s
0	0	514.93507	0	1	512.76440	514.93507	514.93507	0.42%	-	0s
0	0	513.65870	0	1	512.76440	513.65870	513.65870	0.17%	-	0s
0	0	513.02052	0	1	512.76440	513.02052	513.02052	0.05%	-	0s
0	2	513.02052	0	1	512.76440	513.02052	513.02052	0.05%	-	0s

Explored 7 nodes (21 simplex iterations) in 0.07 seconds (0.00 work units)

Thread count was 8 (of 8 available processors)

Solution count 1: 512.764

Optimal solution found (tolerance 1.00e-04)

Best objective 5.127643950055e+02, best bound 5.127656431856e+02, gap 0.0002%

```
[15]: print(rob_obj_val)
```

```
[621.9875788034356, 605.675914901638, 590.1944629255565, 575.4814133891784, 561.4809438720453, 548.1425112478512, 535.4202420009309, 523.2724051268199,
```

```
512.7643950037026, 512.7643950054742]
```

Having a great budget of uncertainty is not always useful because if the linear regression performs well we will prefer to stay close to parameters estimations, hence we have to apply a tradeoff between the conservatism of solution and the revenue we are eager to lose.

See minimum, maximum and average percentage variation between robust optimal solution and Monte Carlo simulation

```
[16]: rob_obj_val = np.array(rob_obj_val)
      perc_var = (sum(obj_val[obj_val < obj_nominal])/sum(obj_val <_
      ↪obj_nominal)-rob_obj_val)/(rob_obj_val)
```

```
[17]: print(perc_var, sum(perc_var)/len(perc_var))
```

```
[-0.1016745 -0.07748139 -0.05328271 -0.02907845 -0.00486862  0.01934678
 0.04356776  0.06779433  0.08967649  0.08967649] 0.004367617825259915
```

```
[ ]:
```